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Competition Between Regulated and Unregulated Generators on Electric Power Networks*

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Abstract

The ongoing restructuring of the electricity sectors in many countries raises the policy question of to what extent the public generating assets should be privatized and what the objective function of any remaining public generation companies should be. We analyze the optimal regulatory policy in the context of a mixed wholesale electricity market in which a private and a public generator engage in Cournot competition. In a standard industry without externalities, instructing the public firm to maximize profits instead of total social surplus under similar settings may turn out to be welfare improving. In an

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electricity market, however, the possibility of congestion in the transmission network and externalities between generators stemming from loop flows change the nature of equilibria and the optimal regulatory policy dramatically. Not only that instruction of pure welfare maximization is not a part of optimal regulatory policy in many instances, instruction of profit maximization is not a part of optimal regulatory policy, except in one case, either. We also extend the literature by allowing the public firm to maximize not only its profits or social welfare, but any convex combination of the two, as well as considering cost asymmetries and shadow cost of public funds. Furthermore, our framework also applies to a wholesale power industry where regulated private firms and unregulated private firms are competing, rather than public and private firms.

1 Introduction

In many countries public ownership of vertically integrated franchise utilities had been the dominant structure of the electricity supply industry prior to the end of 1980's. Since then quite a few countries have witnessed a thorough restructuring in their electricity sectors. With the U.K. leading the way and serving as an example, a number of countries - Australia, New Zealand and Chile, to name a few - disintegrated their formerly vertically integrated franchise utilities, deregulated and opened to competition the generation and the retail segments, while keeping the natural monopoly network segments, i.e. the high-voltage transmission and distribution segments, either under public ownership or under strict regulation. Despite these developments, public ownership of generation assets and capacity is still the predominant type of ownership in many countries; and in many cases where it is not, public production is still significant in the wholesale generation segment. The policy challenge for public authorities in many countries is to what extent the public generating assets should be privatized and what the objective function of any remaining public generation companies should be.

There are quite a few countries with "mixed" wholesale electricity markets in which public generators compete with their private counterparts. Perhaps the most notable examples of mixed wholesale electricity sectors are in the Scandinavian countries. Table 1 displays the breakdown of generation capacity in Sweden, Norway, and Finland into type of ownership. In all three countries, the share of public ownership is large but the share of private ownership is also substantial.[1]

Table 1: Ownership of Generation (%)

	Sweden	Norway	Finland
State	55	25	46
Municipalities/ Counties/Co-ops	20	38	28
Private	25	38	36

(1)

Source: Hjalmarsson [6].

In Sweden, where the largest 10 generators produce more than 90 percent of the electricity that is supplied to retail customers, 50 percent of electricity is

[1] The information cited on electricity industries of Scandinavian countries is from Hjalmarsson [6].

generated by The Swedish State Power Board (Vattenfall). In Norway, where most of the electricity market is served by a large number of small generators, the largest state-owned generator (Statkraft) generates approximately 25 to 30 percent of the total electricity produced. In Finland, the state-owned Imatran Voima (IVO) is the largest generator, providing 40 percent of the total production in the country, amounting to an approximately 45 percent share of the wholesale electricity market.[2]

Profit maximization has typically not been a significant part of the objective of public generation companies. In Sweden, for example, Vattenfall's formal objective has been to break even, with depreciation on replacement values and a rate of interest on loans from the government at the bond rate level being included. Pricing in the wholesale market for bulk power has been indirectly regulated through state ownership of Vattenfall. This has established Vattenfall as a price leader, and yardstick competition between and private generastors has created a downward price pressure. As in Sweden, the formal, government-enforced regulations have historically been fairly weak in Norway and Finland as well. Instead, the industries are to a large extent characterized by publicly owned dominant firm leadership, self-enforced club-regulation, and yardstick competition.

In the U.K., where the electricity sector went through a bold and radical restructuring in the 1990's, the wholesale generation segment was initially intended to be totally privatized. However, the nuclear plants could not be privatized, as risks involved in nuclear power production were perceived to be too high and private investors did not show any interest in acquring them. Thus they were consolidated under one company and kept under public ownership. As of now, British wholesale electricity market is a mixed oligopoly with two private and one public strategic players.[3]

Even in the United States, known as the stronghold of the investor-owned utility model, there is a non-negligible ownership of capacity by the public authorities. Generation capacity owned by the federal government, states, and cooperatives are 9.5 percent, 10.4 percent, and 3.7 percent, respectively, of the overall installed base (Gilbert and Kahn [5]). Gilbert and Kahn [5] argue that the generators owned by the federal government sell much of their

[2] In all of these three Scandinavian countries, especially in Sweden, mixed ownership of single generators is frequent. Therefore, the distinctions between private, municipally owned, and state-owned assets are not as clear-cut as Table 1 may indicate.

[3] For a review of the restructuring process in the U.K. electricity industry, see Newbery and Green [10].

output to utilities owned by states and cooperatives at deep discounts. They estimate that this discount is at least 4 cents/kWh compared to open-market cost for long-term firm power supply, which in turn amounts to a transfer payment of $7 billion annually, or roughly 20% of the combined revenue of publicly-owned and cooperative segments. Analysis of a mixed electricity market can also be tought to apply to a wholesale power industry where regulated private firms and unregulated private firms are competing, rather than public and private firms. Thus, it would also apply to certain states in the U.S. (such as California, New York, New Jersey, Pennsylvania, etc.) where still partially regulated vertically-integrated utilities are competing against unregulated independent generators.

In addition to the countries discussed above, many other countries, developed as well as developing, have already partly privatized and/or totally restructured their electricity industries or are in the process of doing so. It is evident that it will take a long time, if it will ever happen, for all countries to sell all of their public generation assets. Therefore, the wholesale generation segment of electricity industry may remain a mixed oligopoly for many years, with public and private firms operating together.

It is well known from the Industrial Organization literature that in a standard industry structure without externalities, if a public firm compete with private firms in an imperfectly competitive market (i.e. in a 'mixed' market) social welfare might in certain cases be higher when the public firm is instructed to maximize profits instead of maximizing social welfare.[4] The basic intuition behind this result is that when a public firm is instructed to maximize total social surplus, in some cases it tends to produce so much that gains to consumers from high consumption levels are dominated by the allocative inefficiency caused by inefficient public production displacing more efficient private production. Only in a sequential quantity setting game where the public firm is the Stackelberg leader, instructing the public firm to maximize social welfare may indeed lead to higher social welfare.[5]

These results on mixed markets do not cover some of the interesting features that are observed in electricity markets. For example, when two nodes are connected to each other through a transmission line, there might arise congestion on the interface if suppliers to the line demand to use the line

[4]See De Fraja and Delbano [3], [4]. See also Cremer, Marchand, and Thisse [1] for a model in which the public firm is used as an instrument for regulating an oligopolistic market.

[5]See Sertel [11].

over its capacity. That is to say, the interaction between the public and the private firm might be one in which a capacity constraint is involved. To our knowledge this case has not been covered in the existing literature on imperfectly competitive mixed markets. Secondly, when the network involves more than two nodes, e.g. when two different suppliers at two different nodes are connected to the same third node where consumption takes place, certain network externalities will arise. This is due to existence of loop flows in electricity transmission. When different nodes are connected over a transmission network, a trade between two parties can affect a non-participating third party (positively or negatively) by congesting or de-congesting the connecting lines that the third-party uses and thus altering the cost the third party faces or the quantity it will be able to sell at a particular node.[6]

This paper examines whether and under what circumstances the results mentioned above for mixed oligopolies in standard industry structures extend to the wholesale power markets. Both the two-node and the three-node (loop flow) network configurations are considered as alternative power market structures.

We consider an electricity sector that consists of an unregulated private firm and a regulated ("public") firm. We first study a two-node network in which consumers are located at one of the nodes only. One of the producers is located where the demand is, and there is no demand at the other node where the other producer is located. We assume that one of the producers has a cost advantage. So we are considering a situation where a local monopoly, be it a private or a regulated local monopoly, faces competition from a producer, which is more efficient than itself, that is located away from the market. The competition from the far away producer will be limited by the fixed capacity of the transmission line linking the two locations. We consider separately the case where the local monopoly is a private firm as well as the case where the local monopoly is a regulated ("public" firm).

We then study competition between a private generator and a regulated generator in a three-node electricity network. This is the minimum configuration that allows us to analyze the effects of loop flow. Each pair of these nodes is connected by a transmission line with some fixed thermal capacity. The private and the regulated generators are located at two separate nodes and the consumers are located at the third node. There is no demand for

[6] This is because electrons follow a unique path on an electrical transmission network determined by Kirchoff's Law rather than the direction of trade.

power on nodes where the producers are located and there is no generation capacity available in the node where the consumers are located.

The two generators are assumed to choose non-cooperatively and simultaneously the quantitites they generate, i.e there is Cournot type competition betwen them. The quantities they choose are to be thought of as pre-dispatch quantities submitted to an Independent Systems Operator (ISO) that is in charge of running the transmission network. If the chosen quantities by the generators induce power flows that exceed the thermal limit on any part of the network, ISO warns about the infeasibility of the bid dispatch it received and asks the generators to adjust their output. We study the choice of optimal regulatory policy for the regulated firm for a number of different cases.

In Section 2 we introduce the general features of the model we study. In Section 3 we study the two-node (no loop flow) network case. In Section 4 we study the the three-node network case which allows for explicit consideration of loop flows. In Section 5 we summarize our findings and briefly discuss extensions.

2 The Model

We adopt the two simple models of the electricity sector that were studied recently in Joskow and Tirole [7], [8]. The general features of the model that are common in both the two- and the three-node analyses are as follows.

Let P denote the private firm and R denote the regulated firm. The production technologies of the generators are characterized by positive and strictly increasing marginal cost functions. Thus, letting $C_i(q_i)$ denote the cost function of firm i, $C_i' > 0$ and $C_i'' > 0$, $i = P, R$. The consumers' demand for power is characterized by an inverse demand function $P(Q)$. To obtain closed form solutions, we will at the outset adopt the cost function specification

$$C_i(q_i) = \frac{1}{2}c_i q_i^2 + F_i, \tag{2}$$

where $F_i \geq 0$, $i = 1, 2$, is the fixed cost[7], and the linear demand function

[7]Throughout this paper we will assume that the fixed costs are sunk for both firms. That is, we will be concentrating on output decisions in the short run after the firms' generating capacity decisions are already made.

specification

$$P(q_P + q_R) = a - (q_P + q_R). \tag{3}$$

The private firm's objective will be the standard one, i.e. profit maximization. The regulated firm will be assumed to maximize a weighted average of total surplus and its own profits. The total surplus is defined as the sum of consumers' surplus and the total industry profits:

$$W(q_R, q_P) = \int_0^{q_R+q_P} P(Q)\,dQ - P(Q)\,Q + \Pi_P(q_R, q_P) + (1+\eta)\,\Pi_R(q_R, q_P), \tag{4}$$

where $\Pi_i(q_R, q_P)$ is the profit function of firm i, $i = P, R$, $Q \equiv q_R + q_P$, and $\eta \geq 0$ is a parameter reflecting the shadow cost of public funds. The presence of shadow cost of funds indicates that the taxes collected by the government are distortionary, and that each dollar raised and spent by government costs $\$(1+\eta)$ to the society when the (marginal) excess burden of the taxes collected is taken into account.[8]

The regulated firm's objective function is assumed to be

$$\alpha W(q_R, q_P) + (1-\alpha)\Pi_R(q_R, q_P), \tag{5}$$

where $\alpha \in [0,1]$ is the weight attached to total surplus maximization. Thus, the $\alpha = 1$ case refers to a 'pure' public firm whose objective is to maximize total surplus, while in the $\alpha = 0$ case the firm is a pure private firm. The cases where $\alpha \in (0,1)$ trace all possible regulatory regimes between total surplus maximization and profit maximization. An alternative interpretation would be to consider the regulated firm as partially privatized and partially owned by the government, the parameter α indicating the government's share in the firm. In this case the firm's objective function would be assumed to reflect the ownership structure.

3 A Two-Node (No Loop Flow) Network

In this section we analyze the simpler two-node network case where consumers are located at one of the nodes only. Following Joskow and Tirole

[8] Note that the above specification assumes full extraction of the regulated firm's profits by the government.

([7], [8]) we call the node where the consumers are located as the "South". One of the producers is located where the demand is, that is at the South. There is no demand at the other node, namely the "North", where the other producer is located. The producer located away from the consumers, i.e. the producer in the North, will be assumed to have a cost advantage over the producer in the South (the local monopoly).

Below we first analyze the case where the firm in the South is the (profit-maximizing) private firm, and then analyze the case where the regulated firms is located in the South. Since the firm located in the South will have direct access to final consumers it will be assumed to face no capacity constraint in production, while the firm in the North will be constrained by the capacity of the line that connects the South to North. Let K denote the capacity of the line between the North and the South (see Figure 1 for the depiction of the two-node network configuration described).

Firms are assumed to engage in Cournot competition, i.e. they compete by simultaneously choosing output levels, the choice of the output of the firm located in the North being constrained.[9]

3.1 Regulated Firm in the North

In this section we analyze the case where the private firm is located in the South and the regulated firm is located in the North. Thus it will be the regulated firm which faces the capacity constraint K in its output decisions. The marginal cost advantage of the firm in the North, which is the regulated firm in this case, will be represented by assuming $c_R = 1$ and $c_P = c > 1$. The private firm's problem will then be

$$\max_{q_P} \Pi_P (q_R, q_P) = P (q_P + q_R) q_P - \frac{1}{2} c q_P^2 - F_P, \tag{6}$$

implying the response function

$$q_P (q_R) = \frac{a - q_R}{c + 2}. \tag{7}$$

[9] See Borenstein, Bushnell and Stoft [12] for use and justification of Cournot competition in power generation markets. See also Wolak and Patrick [17] for evidence that exercise of market power in the UK has taken place through capacity withholding. Oren [15], Cardell, Hitt and Hogan [13], Smeers and Wei [16], Joskow and Tirole [14] also use Cournot competition in their analysis.

The regulated firm's problem can be written as

$$\begin{array}{ll} \max\limits_{q_R} & \alpha \frac{(q_P+q_R)^2}{2} + \alpha \left[P\left(q_P+q_R\right)q_P - \frac{1}{2}cq_P^2\right] + (1+\alpha\eta)\left[P\left(q_P+q_R\right)q_R - \frac{1}{2}q_R^2\right] \\ \text{s.t.} & q_R \leq K \end{array} \tag{8}$$

The capacity constraint appears in the regulated firm's maximization problem, because, as already mentioned, the ISO in charge of feasible dispatch on the transmission network enforces this constraint. The Kuhn Tucker necessary conditions for the regulated firm's problem are

$$\begin{array}{l} \frac{\partial L}{\partial q_R} = (1+\alpha\eta)\,a - \left[3\left(1+\alpha\eta\right) - \alpha\right] q_R - (1+\alpha\eta)\,q_P - \lambda = 0 \\ \frac{\partial L}{\partial \lambda} \geq 0 \text{ and } \lambda\left[K - q_R\right] = 0 \end{array}$$

Depending on K, the response function of the regulated firm will take a different form. The unconstrained response function of the regulated firm is

$$q_R\left(q_P\right) = \frac{(1+\alpha\eta)\left[a - q_P\right]}{3\left(1+\alpha\eta\right) - \alpha}. \tag{9}$$

Note that $q_R\left(q_P\right)$ above depends on α. Figure 2 depicts the response functions of the two firms for given K. The response function of the regulated firm that passes through point A corresponds to the case when $\alpha = 0$. Let $q_R^N(0)$ be the equilibrium output of the regulated firm when $\alpha = 0$ and K is high enough so that the constraint is not binding for the regulated firm at the equilibrium. Note from (9) that as α increases the regulated firm's response function becomes flatter (in a continuous manner). The response function that passes through point C corresponds to the case when $\alpha = 1$. Let $q_R^N(1)$ be the equilibrium output of the regulated firm when $\alpha = 1$. Note that if $K < q_R^N(0)$, then, regardless of the value of α, the capacity constraint will always be binding; and if $K > q_R^N(1)$, then, the capacity constraint will not be binding for any value of α. For a given level of $K \in \left[q_R^N(0), q_R^N(1)\right]$, there will be an $\alpha = \alpha(K)$ such that the capacity constraint becomes just binding at equilibrium. The response function in Figure 2 that passes through point B shows such a case. For a given K, if $\alpha \leq \alpha(K)$ then the capacity constraint will not be binding at equilibrium, while for $\alpha > \alpha(K)$ it will be binding.

For a given K, suppose that we have $\alpha \leq \alpha(K)$. The (unconstrained) equilibrium levels of output chosen by the regulated and the private firm in this case will be

$$q_R^N(\alpha) = \frac{(c+1)(1+\alpha\eta)a}{(3c+5)(1+\alpha\eta)-(c+2)\alpha}, \tag{10}$$

and

$$q_P^N(\alpha) = \frac{[2(1+\alpha\eta)-\alpha]a}{(3c+5)(1+\alpha\eta)-(c+2)\alpha}, \tag{11}$$

respectively. Note that $q_R^N(\alpha)$ is strictly increasing and $q_P^N(\alpha)$ strictly decreasing in α. The total amount of electricity produced when capacity constraint is not binding will be

$$Q^N(\alpha) = \frac{[(c+3)(1+\alpha\eta)-\alpha]a}{(3c+5)(1+\alpha\eta)-(c+2)\alpha}, \tag{12}$$

and it will be sold at the price

$$P^N(\alpha) = \frac{(c+1)(2(1+\alpha\eta)-\alpha)a}{(3c+5)(1+\alpha\eta)-(c+2)\alpha} \tag{13}$$

Note that $\frac{\partial P^N(\alpha)}{\partial\alpha} < 0$, i.e the (unconstrained) equilibirum price is strictly decreasing in α.

$>$From (10) $\alpha(K)$, defined by $q_R^N(\alpha(K)) = K$, can be computed as

$$\alpha(K) = \frac{(3c+5)K-(c+1)a}{(c+1)\eta a-[(3c+5)\eta-(c+2)]K}. \tag{14}$$

For a given K, suppose now that $\alpha > \alpha(K)$. In this case the capacity constraint will be binding at the equilibrium. In this case the equilibrium output levels will be

$$q_R^B = K \tag{15}$$

and

$$q_P^B = \frac{a-K}{c+2}, \tag{16}$$

for the regulated and the private firm, respectively. The total amount of electricity produced in this case and the consequent market price will be

$$Q^B = \frac{a+(c+1)K}{c+2} \tag{17}$$

and

$$P^B = \frac{(c+1)(a-K)}{c+2}, \tag{18}$$

respectively.

3.1.1 Optimal Regulatory Policy

Note that the equilibrium levels of production computed and stated above are for a given α, which can be viewed as a regulatory policy tool. As mentioned above, the $\alpha = 1$ case will refer to a 'pure' public firm whose objective is to maximize total surplus. A question that arises is whether in this setting it will indeed be optimal to set $\alpha = 1$.

The total surplus in the present case is given by

$$W(q_R, q_P; K, \alpha) = PQ + q_R q_P - \frac{1}{2}(c-1) q_P^2 + \eta \left(P q_R - \frac{1}{2} q_R^2 \right) \tag{19}$$

It will be useful to first find the optimal value of α when the corresponding equilibrium output for the regulated firm turns out to be unconstrained (which, for example, will be the case when K is large enough). Let $W^N(\alpha) = W\left(q_R^N, q_P^N; K, \alpha\right)$, which can be computed by substituting (10), (11), (12), and (13) in (19). Differentiating $W^N(\alpha)$ with respect to α, and checking that the second order conditions are satisfied, results in an optimal value of α given by

$$\alpha^* = \frac{c^2 + 3c + (c+1)\eta}{c^2 + 3c + 1 - (c+1)\eta^2 + 2\eta}. \tag{20}$$

When $c = 1$ and $\eta = 0$, i.e. when the private firm is as efficient as the regulated firm and the shadow cost of public funds is zero, α^* will be equal to $\frac{4}{5}$. For a given η, α^* is increasing in c, and for a given c it is increasing in η. Note that for α^* to fall in the interval $[0,1]$, as per its definition, it has to be the case that $\eta \leq \overline{\eta}$, where $\overline{\eta} = \frac{\left(1-c+\sqrt{c^2+2c+5}\right)}{2(c+1)}$.

Define $K(\alpha) \equiv \alpha^{-1}(K) \equiv q_R^N(\alpha(K))$.[10] For a given α, $K(\alpha)$ is the capacity level which is just equal to the unconstrained equilibrium output of the regulated firm.

If $K \geq K(\alpha^*)$, i.e. if K is high enough so that the equilibrium output of the regulated firm induced by α^* is not constrained, then the optimal regulatory policy will be to set $\alpha = \alpha^*$.

If, on the other hand, $K < K(\alpha^*)$, i.e. if K is not sufficiently high so that the equilibrium output of the regulated firm is constrained at $\alpha = \alpha^*$, then the optimal regulatory policy is to choose $\alpha \in [\alpha(K), 1]$. This is because total surplus is increasing in α for $\alpha < \alpha^*$, and, therefore, it will be optimal to set α at least equal to $\alpha(K)$.

[10] It is easily checked that $\alpha(K)$ is strictly increasing in K.

The following proposition summarizes the optimal regulatory policy when the (more efficient) regulated firm is located in the North.

Proposition 1 *If $K \geq K(\alpha^*)$, then the optimal regulatory policy is to set $\alpha = \alpha^*$. If $K < K(\alpha^*)$, then the optimal regulatory policy is to set $\alpha = \in [\alpha(K), 1]$.*

Observe in (20) that when $c = 1$ and $\eta = 0$, i.e. when the regulated firm and the private firm have the same marginal costs and the shadow cost of public funds is zero, we have $\alpha^* = \frac{4}{5}$. As c or η increases the larger will be the optimal α. As the public firm's efficiency advantage increases the total surplus will increase if the public firm produces more, which a higher α will induce. As for η, a higher η indicates a higher marginal social value of collecting public funds through the profits of the regulated firm, provided that profits of the regulated firm is positive. In that case a higher η will imply a higher optimal α. Note that only in the limiting case where either c or η are very high will the optimal α be equal to 1. In other words, there will be many instances where it will not be optimal to instruct the regulated firm to maximize total surplus. When $K < K(\alpha^*)$, total surplus maximization is not part of an optimal regulatory policy.

3.1.2 The Optimal Choice of Transmission Capacity

The analysis so far considered the capacity level K as fixed. This amounts to treating the capacity cost as sunk cost in an ex-post analysis. Alternatively, one might consider the optimal ex-ante choice of the capacity of the transmission line.

Let $C(K) > 0$ be the total cost of installing a transmission line between the North and the South with a capacity K, where $C'(\cdot) \geq 0$ and $C''(\cdot) > 0$.

As it is costly to install transmission capacity, a total surplus maximizing ex-ante choice of capacity will be the one that leaves no unused capacity in equilibrium. Letting $W^B(K) \equiv W\left(q_R^B, q_P^B; K, \alpha\right)$ be the total surplus at the constrained equilibrium, the optimal ex- ante choice of K will be the solution to the following maximization problem:

$$\max_K W^B(K) - C(K). \tag{21}$$

Note that if $C(K) = 0$, i.e. if capacity could be installed at no cost, then total surplus maximizing level of transmission capacity would be equal to

$K = K(\alpha^*)$. So when $C(K) > 0$ the optimal transmission capacity will be less than $K(\alpha^*)$.

3.2 Regulated Firm in the South

In this section we analyze the case where the private firm is located in the North and the regulated firm is located in the South. It is now the private firm which has the cost advantage. Specifically, it will be assumed that $c_P = 1$ and $c_R = c > 1$. Also, in this case it is the private firm which faces the capacity constraint K in its output decisions. The private firm's problem will therefore be

$$\begin{array}{ll} \max\limits_{q_P} & \Pi_P(q_R, q_P) = P(q_P + q_R) q_P - \frac{1}{2} q_P^2 - F_P \\ \text{s.t.} & q_P \leq K \end{array} \tag{22}$$

The Lagrangian for this constrained optimization problem is

$$L = \quad P(q_P + q_R) q_P - \tfrac{1}{2} q_P^2 - F_P + \lambda [K - q_P]$$

The Kuhn Tucker necessary conditions are

$$\begin{array}{l} \frac{\partial L}{\partial q_P} = a - 3q_P - q_R - \lambda = 0 \\ \frac{\partial L}{\partial \lambda} \geq 0 \text{ and } \lambda [K - q_P] = 0 \end{array}$$

Depending on K, the response function of the private firm will take a different form. The unconstrained response function of the private firm is

$$q_P(q_R) = \frac{a - q_R}{3}. \tag{23}$$

The regulated firm's problem in this case is

$$\max_{q_R} \quad \alpha \tfrac{(q_P + q_R)^2}{2} + \alpha \left[P(q_P + q_R) q_P - \tfrac{1}{2} q_P^2 \right] + (1 + \alpha\eta) \left[P(q_P + q_R) q_R - \tfrac{1}{2} c q_R^2 \right], \tag{24}$$

implying the response function

$$q_R(q_P) = \frac{(1 + \alpha\eta)[a - q_P]}{(2 + c)(1 + \alpha\eta) - \alpha}. \tag{25}$$

Note that, as in the previous case, the response function of the regulated firm depends on α. Figure 3 shows the response functions of the two firms for given K. The response function for the regulated firm that passes through A corresponds to the case when $\alpha = 0$. Let $\widetilde{q}_P^N(0)$ be the equilibrium output of the private firm when $\alpha = 0$ and K is high enough so that the constraint is not binding for the private firm at the equilibrium. Observe from (25) that as α increases the regulated firm's response function becomes flatter (in a continuous manner). The response function that passes through point C corresponds to the case when $\alpha = 1$. Let $\widetilde{q}_P^N(1)$ be the equilibrium output of the private firm when $\alpha = 1$ and K is high enough so that the constraint is not binding for the private firm at the equilibrium. Note that we will have $\widetilde{q}_P^N(0) > \widetilde{q}_P^N(1)$. If $K < \widetilde{q}_R^N(1)$, then, regardless of the value of α, the capacity constraint will always be binding for the private firm; and if $K \geq \widetilde{q}_R^N(0)$, then the capacity constraint will not be binding for any value of α. For a given level of $K \in \left[\widetilde{q}_P^N(1), \widetilde{q}_P^N(0)\right]$, there will be an $\alpha = \widetilde{\alpha}(K)$ such that the capacity constraint becomes just binding for the private firm at equilibrium. The response function in Figure 3 that passes through point B shows such a case. For a given K, if $\alpha \geq \widetilde{\alpha}(K)$ then the capacity constraint will not be binding at equilibrium, while for $\alpha \leq \widetilde{\alpha}(K)$ it will be binding.

For a given K, suppose now that $\alpha \geq \widetilde{\alpha}(K)$. The (unconstrained) equilibrium levels of output chosen by the regulated and the private firm in this case will be

$$\widetilde{q}_R^N(\alpha) = \frac{2(1+\alpha\eta)a}{(3c+5)(1+\alpha\eta) - 3\alpha}, \tag{26}$$

and

$$\widetilde{q}_P^N(\alpha) = \frac{[(c+1)(1+\alpha\eta) - \alpha]a}{(3c+5)(1+\alpha\eta) - 3\alpha}, \tag{27}$$

respectively. Note that, as in the previous case, $\widetilde{q}_R^N(\alpha)$ is strictly increasing and $\widetilde{q}_P^N(\alpha)$ is decreasing in α. It can be checked from (26) and (27) that $\widetilde{q}_R^N(\alpha) > \widetilde{q}_P^N(\alpha)$ iff $c - 1 < \frac{\alpha}{1+\alpha\eta}$. This condition will always hold if $\alpha = 0$, i.e. if both firms are private, the more efficient one will produce more, regardless of the value of η. When $\alpha \neq 0$, however, the regulated firm may end up producing more even though it is less efficient. For a given α,the regulated firm's equilibrium output will be higher the less its relative cost inefficiency or the shadow cost of public funds is. The total amount of electricity produced when capacity constraint is not binding will be

$$\widetilde{Q}^N(\alpha) = \frac{[(c+3)(1+\alpha\eta) - \alpha]a}{(3c+5)(1+\alpha\eta) - 3\alpha}, \tag{28}$$

and it will be sold at the price

$$\widetilde{P}^N(\alpha) = \frac{2(c+1)\left[(1+\alpha\eta)-\alpha\right]a}{(3c+5)(1+\alpha\eta)-3\alpha} \tag{29}$$

>From (26) $\widetilde{\alpha}(K)$, which is given by $\widetilde{q}_P^N(\widetilde{\alpha}(K)) = K$, can be computed as

$$\widetilde{\alpha}(K) = \frac{(3c+5)K-(c+1)a}{\left[(c+1)\eta-1\right]a-\left[(3c+5)\eta-3\right]K}. \tag{30}$$

For a given K, suppose now that $\alpha < \widetilde{\alpha}(K)$. In this case the capacity constraint will be binding for the private firm at the equilibrium. In this case the equilibrium output levels will be

$$\widetilde{q}_P^B = K \tag{31}$$

and

$$\widetilde{q}_R^B = \frac{(1+\alpha\eta)(a-K)}{(c+2)(1+\alpha\eta)-\alpha}, \tag{32}$$

for the private and the regulated firm, respectively. The total amount of electricity produced in this case and the consequent market price will be

$$\widetilde{Q}^B = \frac{(1+\alpha\eta)a+\left[(1+\alpha\eta)(c+1)-\alpha\right]K}{(c+2)(1+\alpha\eta)-\alpha} \tag{33}$$

and

$$\widetilde{P}^B = \frac{\left[(1+\alpha\eta)(c+1)-\alpha\right](a-K)}{(c+2)(1+\alpha\eta)-\alpha}, \tag{34}$$

respectively.

3.2.1 Optimal Regulatory Policy

As was done in the case where the regulated firm is located in the North, the problem of optimal choice of α, which was viewed as the regulatory policy tool, will be examined in the present case. Note that in the former case the firm which was located away from the consumers, and thus was subject to capacity constraint, was under regulation. Furthermore the regulated firm was assumed to be the more efficient firm. In the current case the regulated firm is located where the consumers are and the issue now is to see how competition from a more efficient private firm affects outcomes and the optimal regulatory policy.

The expression for the total surplus in this case is given by

$$\widetilde{W}(q_R, q_P; K, \alpha) = PQ + q_R q_P + \eta \left[P q_R - \frac{(1+\eta)c - 1}{2\eta} q_R^2 \right] \tag{35}$$

As was done in the previous section, the optimal value of α when the corresponding equilibrium output for the private firm will be unconstrained will be a useful benchmark. Let $\widetilde{W}^N(\alpha) = \widetilde{W}\left(\widetilde{q}_R^N, \widetilde{q}_P^N; K, \alpha\right)$, which is computed by substituting (26), (27), (28) and (29) in (35). Differentiating $\widetilde{W}^N(\alpha)$ with respect to α, and checking that the second order conditions are satisfied, results in an optimal value of α given by

$$\widetilde{\alpha}^{*N} = \frac{5 + 2\eta - c}{5 + (c+1)\eta - 2\eta^2} \tag{36}$$

Note that for $\widetilde{\alpha}^{*N} \in [0, 1]$, as per its definition, it has to be the case that the shadow cost of public funds $\eta \leq \widetilde{\eta}$, where $\widetilde{\eta} = \frac{\left(c - 1 + \sqrt{c^2 + 6c + 1}\right)}{4}$, we will have $\widetilde{\alpha}^{*N} \in \left[0, \frac{4}{5}\right]$. Note that $\widetilde{\alpha}^{*N}$ is increasing in η.

Define $\widetilde{K}(\alpha) \equiv \widetilde{\alpha}^{-1}(K) \equiv \widetilde{q}_P^N(\widetilde{\alpha}(K))$. If $K \geq \widetilde{K}\left(\widetilde{\alpha}^{*N}\right)$, i.e. if K is high enough so that the equilibrium output of the private firm induced by $\widetilde{\alpha}^{*N}$ is not constrained, then the optimal regulatory policy will be to set $\alpha = \widetilde{\alpha}^{*N}$.

If, on the other hand, $K < \widetilde{K}\left(\widetilde{\alpha}^{*N}\right)$, then the equilibrium output of the private firm will be constrained when $\alpha = \widetilde{\alpha}^{*N}$. In the previous section where the regulated firm was more efficient but subject to a capacity constraint, the policy tool α no more affected the equilibrium outcome once the capacity constraint became binding. In the current case, where now the regulated firm is less efficient but is not subject to a capacity constraint, α will continue to affect the equilibrium outcome even when the private firm's output is constrained. Let $\widetilde{W}^B(\alpha) \equiv \widetilde{W}\left(\widetilde{q}_R^B, \widetilde{q}_P^B; K, \alpha\right)$, which is computed by substituting (31), (32), (33) and (34) in (35). Differentiating $\widetilde{W}^B(\alpha)$ with respect to α gives

$$\frac{\partial \widetilde{W}^B(\alpha)}{\partial \alpha} = \frac{(a-K)^2 (1-\alpha)}{\left[(c+2)(1+\alpha\eta) - \alpha\right]^3}, \tag{37}$$

which implies that the optimal choice for α is 1.[11] Note, however that $\widetilde{q}_P^N(\alpha)$ is decreasing in α, and when α is equal to 1 the constraint may no longer be

[11]It can be checked that $\frac{\partial^2 \widetilde{W}^B(\alpha)}{\partial \alpha^2}$ evaluated at $\widehat{\widetilde{\alpha}}^B = 1$ is strictly less than zero for all parameter values.

binding. Therefore, if $K > \widetilde{q}_P^N(1) = K(1)$ then it will be optimal to set $\alpha = \widetilde{\alpha}(K)$. If, on the other hand, $K \leq \widetilde{q}_P^N(1) = K(1)$ then is the optimal policy choice in this case is to set α equal to 1

The following proposition summarizes the optimal regulatory policy when the less efficient regulated firm is located where the consumers are and faces competition from a more efficient private firm connected to the demand center with a transmission line of given capacity K.

Proposition 2 *If $K \geq \widetilde{K}\left(\widetilde{\alpha}^{*N}\right)$, then the optimal regulatory policy is to set $\alpha = \widetilde{\alpha}^{*N}$. If $K < \widetilde{K}\left(\widetilde{\alpha}^{*N}\right)$, then the optimal policy is to set $\alpha = \widetilde{\alpha}^{*B}$, where*

$$\widetilde{\alpha}^{*B} = \begin{cases} \widetilde{\alpha}(K) & \text{if } K > \widetilde{q}_P^N(1) \\ 1 & \text{otherwise} \end{cases} \tag{38}$$

equal to 1.

Proposition 2 shows that if capacity level is not high enough, it will be optimal to set α equal to 1, i.e. to instruct the regulated firm to maximize total surplus. For certain values of the cost and demand parameters it turns out to be optimal to have the regulated firm produce as much as possible despite its relative cost inefficiency.

3.2.2 The Optimal Choice of Transmission Capacity

The analysis of ex-ante optimal choice of capacity level K of the transmission line in this case is exactly the same as in the previous case. As it is costly to install transmission capacity, a total surplus maximizing ex-ante choice of capacity will be the one that leaves no unused capacity in equilibrium. Letting $\widetilde{W}^B(K) \equiv W\left(\widetilde{q}_R^B, \widetilde{q}_P^B; K, \alpha\right)$ be the total surplus at the constrained equilibrium, the optimal ex- ante choice of K will be the solution to the following maximization problem:

$$\max_K \widetilde{W}^B(K) - C(K), \tag{39}$$

where $C(K) > 0$ is the total cost of installing a transmission line with capacity K, with $C'(\cdot) \geq 0$ and $C''(\cdot) > 0$. similar to the analysis of the previous section, if $C(K) = 0$, i.e. if capacity could be installed at no cost, then total surplus maximizing level of transmission capacity would be equal to $K = K(\widetilde{\alpha}^*)$. So when $C(K) > 0$ the optimal transmission capacity will be less than $K(\widetilde{\alpha}^*)$.

4 Three-Node Network

In this section we analyze the more general three-node case. In the case of multiple interconnected links, the networks exhibit what are called *loop flows*, which are an essential characteristic of electricity networks. For example, in a three-node network, electricity sent from one node to the other not only affects the line connecting these two nodes, but also affects the congestion on the other two lines.

As in Joskow and Tirole [7], [8], we study a simple three-node network with two nodes of production (the private firm located at one and the regulated firm at the other) and one node of consumption. We consider the case where the transmission line between two generation nodes is capacity constrained (see Figure 4).

4.1 The Regulated Firm is More Efficient

We first consider the case when the regulated firm is more efficient, i.e. $c_R = 1$ and $c_P = c > 1$. The response functions of the private and the regulated firm in this case, given by

$$q_P(q_R) = \frac{a - q_R}{c + 2}, \tag{40}$$

and

$$q_R(q_P) = \frac{(1 + \alpha\eta)[a - q_P]}{3(1 + \alpha\eta) - \alpha}, \tag{41}$$

respectively, are the same as in Section 3.1.[12] As before, the response function of the regulated firm $q_R(q_P)$ depends on α, which will change the nature of equilibria attained. Figure 5 displays the response functions of the two firms for given K. The response function of the regulated firm becomes flatter (in a continuous manner) as α increases. The response function that passes through point A in Figure 5 corresponds to the case when $\alpha = 0$, and the one that passes through point C corresponds to the case when $\alpha = 1$.

The line between the two generators has capacity K. Assuming that there are no losses on the lines, we will have $q_P + q_R$ equal the quantity demanded at the consumption node. Electricity flowing from the generators

[12] The objective functions of the private and the regulated firm are as in Section 3.1 (expressions (6) and (8), respectively).

to the consumers must follow the path of least resistance. This, in our case, translates into the following constraint on q_P, the amount of electricity produced by the private firm, and q_R, the amount produced by the regulated firm:

$$\left|\frac{q_R}{3} - \frac{q_P}{3}\right| \leq K. \tag{42}$$

Given that in this case we are considering a more efficient regulated firm, which in equilibrium will lead to a higher output for the regulated firm than that of the private one, this constraint becomes

$$q_R - q_P \leq 3K. \tag{43}$$

Let $\overline{q}_R^N(\alpha)$ and $\overline{q}_P^N(\alpha)$ be the unconstrained equilibrium output level of the regulated and the private firm for a given α. So $\overline{K}(0) > 0$ will be the capacity level such that $\overline{q}_R^N(0) - \overline{q}_P^N(0) = 3\overline{K}(0)$. Similarly, let $\overline{K}(1) > 0$ be the capacity level such that $\overline{q}_R^N(1) - \overline{q}_P^N(1) = 3\overline{K}(1)$. Figure 5 depicts the capacity constraint for both $K = \overline{K}(0)$ and $K = \overline{K}(1)$. For a given level of $K \in \left[\overline{K}(0), \overline{K}(1)\right]$, there will be an $\alpha = \overline{\alpha}(K)$ such that the capacity constraint $q_R - q_P = 3K$ is just binding in equilibrium. In Figure 5, the capacity constraint line $q_R - q_P = 3K$ and the corresponding response function for the regulated firm both of which passes through point B show such a case. For a given K, if $\alpha \leq \overline{\alpha}(K)$ then the capacity constraint will not be binding, while for $\alpha > \overline{\alpha}(K)$ it will be binding. Observe that if $K < \overline{K}(0)$ then the capacity constraint will be binding no matter what α is, and if $K \geq \overline{K}(1)$ then it will not be binding no matter what α is.

For a given K, suppose that we have $\alpha \leq \overline{\alpha}(K)$. The (unconstrained) equilibrium output choices by the regulated and the private firm at will be

$$\overline{q}_R^N(\alpha) = \frac{(c+1)(1+\alpha\eta)\,a}{(3c+5)(1+\alpha\eta) - (c+2)\alpha} \tag{44}$$

and

$$\overline{q}_P^N(\alpha) = \frac{[2(1+\alpha\eta) - \alpha]\,a}{(3c+5)(1+\alpha\eta) - (c+2)\alpha}, \tag{45}$$

respectively. Note that $\overline{q}_R^N(\alpha) > \overline{q}_P^N(\alpha)$. The corresponding total output and price levels are

$$\overline{Q}^N(\alpha) = \frac{[(c+3)(1+\alpha\eta) - \alpha]\,a}{(3c+5)(1+\alpha\eta) - (c+2)\alpha} \tag{46}$$

and

$$\overline{P}^{N}(\alpha) = \frac{(c+1)(2(1+\alpha\eta)-\alpha)\,a}{(3c+5)(1+\alpha\eta)-(c+2)\alpha}. \tag{47}$$

Note that $\overline{\alpha}(K)$ can be computed from expressions (44) and (45) as

$$\overline{\alpha}(K) = \frac{3(3c+5)K-(c-1)a}{(c-1)\eta a+1-3[(3c+5)\eta-(c+2)]K}. \tag{48}$$

We will confine our attention to the space of parameters so that $\overline{\alpha}(K) \in [0,1]$ for all K.

For a given K, suppose now that $\alpha > \overline{\alpha}(K)$. Take, for example, $\alpha = 1$ and consider the capacity level K shown in Figure 5. The unconstrained equilibrium which would occur at point C is not attainable in this case. The constrained equilibirium will occur at point B (at the intersection of the private firm's response function and the capacity constraint equation $q_R - q_P = 3K$).

When the capacity constraint is binding the equilibrium level of output produced by the regulated and the private firm will be

$$\overline{q}_R^B = \frac{a+3(c+2)K}{c+3} \tag{49}$$

and

$$\overline{q}_P^B = \frac{a-3K}{c+3}, \tag{50}$$

respectively, leading to the total output and price given by

$$\overline{Q}^B = \frac{2a+3(c+1)K}{c+3}$$

and

$$\overline{P}^B = \frac{(c+1)(a-3K)}{c+3}.$$

We will assume, as before, that there is an Independent System Operator, tasked with ensuring the stability of the network, which will strictly enforce the capacity constraint in cases where it is strictly binding in equilibrium.

4.1.1 Optimal Regulatory Policy

Expression (19) also gives the total surplus for the current case. As in the two-node case, we first consider the optimal choice of the policy variable α when the capacity constraint is not binding at equilibrium. In fact, observe that the welfare analysis of the three-node case when the capacity constraint is not binding is going to be exactly the same as the welfare analysis of the unconstrained two-node case. Without the capacity constraint the features of the two-node and the three-node network configurations we study become identical. Thus, the optimal regulatory policy in the three-node network when the regulated firm is more efficient and the capacity of the line connecting the two generators is large enough will be

$$\overline{\alpha}^* = \frac{c^2 + 3c + \eta\,(c+1)}{c^2 + 3c + 1 - \eta^2\,(c+1) + 2\eta} \in \left[\frac{4}{5}, 1\right], \tag{51}$$

as given in expression (20) of Section 3.1.1 where we studied the corresponding two-node case. Note again that for $\overline{\alpha}^*$ to fall in the interval $[0,1]$, as per its definition, it has to be the case that $\eta \leq \overline{\eta}$, where $\overline{\eta} = \frac{\left(1-c+\sqrt{c^2+2c+5}\right)}{2(c+1)}$.

Let $\overline{K}\,(\alpha) \equiv \overline{\alpha}^{-1}\,(K) \equiv \frac{1}{3}\left[\overline{q}_R^N\,(\overline{\alpha}\,(K)) - \overline{q}_P^N\,(\overline{\alpha}\,(K))\right]$. For a given α, $\overline{K}\,(\alpha)$ is the capacity level which is just binding at the unconstrained equilibrium, and it can be computed by using the expression (48).

If $K \geq \overline{K}\,(\overline{\alpha}^*)$, i.e. if K is high enough so that the equilibrium output of the regulated firm induced by $\overline{\alpha}^*$ is not constrained, then the optimal regulatory policy will be to set $\alpha = \overline{\alpha}^*$.

If, on the other hand, $K < \overline{K}\,(\overline{\alpha}^*)$, i.e. if K is not sufficiently high so that the equilibrium output of the regulated firm is constrained at $\alpha = \overline{\alpha}^*$, then the optimal regulatory policy will be to choose $\alpha = \overline{\alpha}\,(K)$. Since total surplus is strictly increasing in α for $\alpha < \overline{\alpha}^*$, it will be optimal to set α at least equal to $\overline{\alpha}\,(K)$. Setting $\alpha > \overline{\alpha}\,(K)$ would lead jeopardizing safety of the transmission network as it would lead to the violation of the capacity constraint on the line connecting the two generators.

The following proposition summarizes the optimal regulatory policy when there is competition in a three-node network with transmission capacity constraint between a private generator and a more efficient regulated generator.

Proposition 3 *If $K \geq \overline{K}\,(\overline{\alpha}^*)$, then the optimal regulatory policy is to set $\alpha = \overline{\alpha}^*$. If $K < \overline{K}\,(\overline{\alpha}^*)$, then the optimal regulatory policy is to set $\alpha = \overline{\alpha}\,(K)$.*

Proposition 3 says that when α is set at its optimal value $\overline{\alpha}^*$, there will a capacity level $K = \overline{K}^*$ such that $q_R^N(\overline{\alpha}^*) - q_P^N(\overline{\alpha}^*) = 3\overline{K}^*$, i.e. a capacity level that will be just binding at the optimal value of α. As in the two-node case, since $\overline{\alpha}^* \in \left[\frac{4}{5}, 1\right]$, the optimal policy will not involve total surplus maximization when the network is not congested. When the capacity of the network is low so that the network will be congested in equilibrium, the optimal regulatory policy will again not involve total surplus maximization.

4.1.2 The Optimal Choice of Transmission Capacity

Noting once more that the total surplus maximizing ex-ante choice of capacity will be the one that leaves no unused capacity in equilibrium, the optimal ex-ante optimal choice of K will be the solution to the following maximization problem:

$$\max_K \overline{W}^B(K) - C(K), \tag{52}$$

where $\overline{W}^B(K) \equiv W\left(\overline{q}_R^B, \overline{q}_P^B; K, \alpha\right)$ and $C(K) > 0$ is the total cost of installing a transmission line with capacity K, with $C'(\cdot) \geq 0$ and $C''(\cdot) > 0$. If $C(K) = 0$, i.e. if capacity could be installed at no cost, then total surplus maximizing level of transmission capacity would be equal to $K = K(\overline{\alpha}^*)$. So when $C(K) > 0$ the optimal transmission capacity will be less than $K(\overline{\alpha}^*)$.

4.2 The Private Firm is More Efficient

We now look at the case where the private firm is more efficient, i.e. $c_P = 1$ and $c_R = c > 1$. The response functions of the private and the regulated firm, given by

$$q_P(q_R) = \frac{a - q_R}{3}. \tag{53}$$

and

$$q_R(q_P) = \frac{(1 + \alpha\eta)[a - q_P]}{(2 + c)(1 + \alpha\eta) - \alpha} \tag{54}$$

respectively, are the same as in Section 3.2.[13] As before, the response function of the regulated firm $q_R(q_P)$ depends on α, which will change the nature of equilibria attained. Figure 6 shows the response functions of the two

[13] The objective functions of the private and the regulated firm will be as in Section 3.1 (expressions (22) and (24), respectively).

firms for given K. As in Figure 5, the regulated firm's response functions passing through points A and C correspond to the cases of $\alpha = 0$ and $\alpha = 1$, respectively. The response function of the regulated firm gets flatter as α increases from 0 to 1, i.e. the more welfare conscious the regulated firm is the more responsive it is to changes in the private firm's output.

Note from Figure 6 that, unlike the previous case in which the more efficient regulated firm always produced more than the private firm in unconstrained equilibirum, in this case the unconstrained equilibrium output of the regulated firm may be greater or smaller than that of the private firm. When $\alpha = 0$, i.e. when both firms are profit-maximizing private firms, the unconstrained equilibrium will always be above the $45°$ line in Figure 6, as a consequence of the fact that a more efficient private firm will always produce more than a less efficient private firm at the equilibirum of a quantitiy setting game. When $\alpha = 1$, the unconstrained equilibirium may occur above or below the $45°$ line depending on values of the parameters.

Let $\widetilde{\overline{q}}_R^N(0)$ and $\widetilde{\overline{q}}_R^N(0)$ be the unconstrained equilibrium output level of the regulated and the private firm, respectively, when $\alpha = 0$. Let $\widetilde{\overline{K}}(0) > 0$ be the capacity level such that $\widetilde{\overline{q}}_P^N(0) - \widetilde{\overline{q}}_R^N(0) = 3\overline{K}(0)$ and let $\widetilde{\overline{K}}(1) > 0$ be the capacity level such that $\left|\widetilde{\overline{q}}_P^N(1) - \widetilde{\overline{q}}_R^N(1)\right| = 3\widetilde{\overline{K}}(1)$.[14] A capacity constraint line that passes through point A in Figure 6 would correspond to the capacity level $\overline{K}(0)$ and one that passes through point C would correspond to the capacity level $\widetilde{\overline{K}}(1)$. If $K > \widetilde{\overline{K}}(0)$, then the capacity constraint is not binding for any value of α; and if $K < \frac{1}{3}\left|\widetilde{\overline{q}}_P^N(1) - \widetilde{\overline{q}}_R^N(1)\right|$, then the capacity constraint will be binding for any value of α. For a given level of $K \in \left[\widetilde{\overline{K}}(1), \widetilde{\overline{K}}(0)\right]$, there will be two values for α, $\widetilde{\overline{\alpha}}_1(K)$ and $\widetilde{\overline{\alpha}}_2(K)$, both of which will make the capacity constraint $|q_P - q_R| = 3K$ just binding. In Figure 6, point B and B' correspond to equilibria at two such just values of α. Let $\widetilde{\overline{\alpha}}_1(K)$ be the value of α that results in point B being the equilibrium outcome, and let $\widetilde{\overline{\alpha}}_2(K)$ be the value of α that results in point B' being the equilibrium outcome. Observe that $\widetilde{\overline{q}}_P^N\left(\widetilde{\overline{\alpha}}_1(K)\right) > \widetilde{\overline{q}}_R^N\left(\widetilde{\overline{\alpha}}_1(K)\right)$ and $\widetilde{\overline{q}}_R^N\left(\widetilde{\overline{\alpha}}_2(K)\right) > \widetilde{\overline{q}}_P^N\left(\widetilde{\overline{\alpha}}_2(K)\right)$ so that, for the case depicted in Figure 6, we have $\widetilde{\overline{\alpha}}_2(K) > \widetilde{\overline{\alpha}}_1(K)$.

For a given K, suppose that we have $\alpha \in \left[\widetilde{\overline{\alpha}}_1(K), \widetilde{\overline{\alpha}}_2(K)\right]$. In this case the equilibirum output choices of the regulated and the private firm will be

[14] >From previous paragraph we know that $\widetilde{\overline{q}}_P^N(0) > \widetilde{\overline{q}}_R^N(0)$.

unconstrained and they are given by

$$\tilde{\tilde{q}}_R^N = \frac{2(1+\alpha\eta)a}{(3c+5)(1+\alpha\eta)-3\alpha} \tag{55}$$

and

$$\tilde{\tilde{q}}_P^N = \frac{[(c+1)(1+\alpha\eta)-\alpha]a}{(3c+5)(1+\alpha\eta)-3\alpha}, \tag{56}$$

respectively, leading to total output and price levels of

$$\tilde{\tilde{Q}}^N = \frac{[(c+3)(1+\alpha\eta)-\alpha]a}{(3c+5)(1+\alpha\eta)-3\alpha} \tag{57}$$

and

$$\tilde{\tilde{P}}^N = \frac{2[(c+1)(1+\alpha\eta)-\alpha]a}{(3c+5)(1+\alpha\eta)-3\alpha}. \tag{58}$$

It can be easily checked from expressions (55) and (56) that $\tilde{\tilde{q}}_P^N \geq \tilde{\tilde{q}}_R^N$ iff $c-1 \geq \frac{\alpha}{1+\alpha\eta}$. Note that $\tilde{\tilde{\alpha}}(K) \in \left\{\tilde{\tilde{\alpha}}_1(K), \tilde{\tilde{\alpha}}_2(K)\right\}$ can be computed from expressions (55) and (56) as

$$\tilde{\tilde{\alpha}}(K) = \left|\frac{3(3c+5)K-(c-1)a}{(c-1)\eta a-1-3[(3c+5)\eta-3)]K}\right|. \tag{59}$$

Suppose now that either $\alpha < \tilde{\tilde{\alpha}}_1(K)$ or $\alpha > \tilde{\tilde{\alpha}}_2(K)$. In either of these cases the capacity constraint will be binding. When $c-1 \geq \frac{\alpha}{1+\alpha\eta}$ so that $\tilde{\tilde{q}}_P^N \geq \tilde{\tilde{q}}_R^N$, the resulting equilibrium level of output produced by the regulated and the private firm will be

$$\tilde{\tilde{q}}_R^B = \frac{(1+\alpha\eta)(a-3K)}{(1+\alpha\eta)(c+3)-\alpha} \tag{60}$$

and

$$\tilde{\tilde{q}}_P^B = \frac{(1+\alpha\eta)a+3K[(1+\alpha\eta)(c+2)-\alpha]}{(1+\alpha\eta)(c+3)-\alpha}, \tag{61}$$

respectively, leading to the total output and price given by

$$\tilde{\tilde{Q}}^B = \frac{2a(1+\alpha\eta)+3K[(1+\alpha\eta)(c+1)-\alpha]}{(1+\alpha\eta)(c+3)-\alpha} \tag{62}$$

and

$$\widetilde{\widetilde{P}}^B = \frac{[(1+\alpha\eta)(c+1)-\alpha](a-3K)}{(1+\alpha\eta)(c+3)-\alpha}. \tag{63}$$

When, on the other hand, $c-1 < \frac{\alpha}{1+\alpha\eta}$ so that $\widetilde{\widetilde{q}}_P^N < \widetilde{\widetilde{q}}_R^N$, the resulting equilibrium level of output produced by the regulated and the private firm will be

$$\widetilde{\widetilde{q}}_R^B = \frac{\alpha+9K}{4} \tag{64}$$

and

$$\widetilde{\widetilde{q}}_P^B = \frac{a-3K}{4}, \tag{65}$$

respectively, leading to the total output and price given by

$$\widetilde{\widetilde{Q}}^B = \frac{a+3K}{2} \tag{66}$$

and

$$\widetilde{\widetilde{P}}^B = \frac{a-3K}{2}. \tag{67}$$

4.2.1 Optimal Regulatory Policy

The total surplus for the current case is given by the expression (35). As in the previous cases, we first consider the choice of the policy variable α when the capacity constraint is not binding at equilibrium, which provides a useful benchmark in treating cases where the constraint are binding. Following the same steps as in optimal policy analyses in previous sections, the optimal regulatory policy in the three-node network when the private firm is more efficient and the capacity of the line connecting the two generators is large enough is to set α equal to

$$\widetilde{\widetilde{\alpha}}^* = \frac{5+2\eta-c}{5+(c+1)\eta-2\eta^2} \in \left[0, \frac{4}{5}\right]. \tag{68}$$

Note that this is the same as expression (36) of Section 3.2.1 where we studied the corresponding two-node case. In order that we have $\widetilde{\widetilde{\alpha}}^* \in [0,1]$, as per its definition, we have to have $\eta \leq \widetilde{\eta}$, where $\widetilde{\eta} = \frac{\left(c-1+\sqrt{c^2+6c+1}\right)}{4}$. Note that $\widetilde{\widetilde{\alpha}}^*$ is increasing in η.

Let $\widetilde{\widetilde{K}}(\alpha) \equiv \widetilde{\widetilde{\alpha}}^{-1}(K) \equiv \frac{1}{3}\left|\overline{q}_R^N(\overline{\alpha}(K)) - \overline{q}_P^N(\overline{\alpha}(K))\right|$. For a given α, $\widetilde{\widetilde{K}}(\alpha)$ is the capacity level which is just binding at the unconstrained equilibrium, and it can be computed by using the expression (59).

If $K \geq \widetilde{\widetilde{\overline{K}}}\left(\widetilde{\widetilde{\alpha}}^*\right)$, i.e. if K is high enough so that the equilibrium output of the regulated firm induced by $\widetilde{\widetilde{\alpha}}^*$ is not constrained, then the optimal regulatory policy will be to set $\alpha = \widetilde{\widetilde{\alpha}}^*$.

If, on the other hand, $K < \overline{K}\left(\widetilde{\widetilde{\alpha}}^*\right)$, i.e. if K is not sufficiently high so that the equilibrium output of the regulated firm is constrained at $\alpha = \widetilde{\widetilde{\alpha}}^*$, then the optimal regulatory policy will depend on the sign of $\left|\widetilde{\widetilde{q}}_P^N - \widetilde{\widetilde{q}}_R^N\right|$. If $c - 1 \geq \frac{\alpha}{1+\alpha\eta}$ so that $\widetilde{\widetilde{q}}_P^N \geq \widetilde{\widetilde{q}}_R^N$, then the total surplus will be computed using the expressions (60), (61), (62), and (63). If $c - 1 < \frac{\alpha}{1+\alpha\eta}$ so that $\widetilde{\widetilde{q}}_P^N < \widetilde{\widetilde{q}}_R^N$, then, equilibirum point being the intersection of the private firm's reaction function and and the capacity constraint line, the total surplus will be computed using expressions (64), (65), (66), and (67).

The proposition below summarizes the optimal regulatory policy in when $K < \widetilde{\widetilde{\overline{K}}}\left(\widetilde{\widetilde{\alpha}}^*\right)$, i.e. when K is not sufficiently high so that the equilibrium output of the regulated firm is constrained at $\alpha = \widetilde{\widetilde{\alpha}}^*$.

Proposition 4 *If* $K \geq \widetilde{\widetilde{\overline{K}}}\left(\widetilde{\widetilde{\alpha}}^*\right)$, *then the optimal regulatory policy is to set* $\alpha = \widetilde{\widetilde{\alpha}}^*$. *If* $K < \widetilde{\widetilde{\overline{K}}}\left(\widetilde{\widetilde{\alpha}}^*\right)$, *then the optimal regulatory policy variable* α *belongs to the set* $\left\{0, \frac{a(c+1)-6K(c+2)}{2a-9K-\eta[a(c+1)-6K(c+2)]}, 1\right\}$ *when* $c - 1 \geq \frac{\alpha}{1+\alpha\eta}$, *and the optimal regulatory policy is to set* α *equal to* $\widetilde{\widetilde{\alpha}}(K)$ *when* $c - 1 < \frac{\alpha}{1+\alpha\eta}$.

4.2.2 The Optimal Choice of Transmission Capacity

The total surplus maximizing ex-ante choice of capacity in this case will be the solution to the following maximization problem:

$$\max_K \widetilde{\widetilde{\overline{W}}}^B(K) - C(K), \tag{69}$$

where $\widetilde{\widetilde{\overline{W}}}^B(K) \equiv W\left(\widetilde{\widetilde{q}}_R^B, \widetilde{\widetilde{q}}_P^B; K, \alpha\right)$ and $C(K) > 0$ is the total cost of installing a transmission line with capacity K, with $C'(\cdot) \geq 0$ and $C''(\cdot) > 0$. In contrast to the previous sections, the optimal ex-ante choice of K in this case depends on whether at the constrained equilibrium it will be total surplus maximizing to have the regulated firm produce more than the private firm or not. If $\widetilde{\widetilde{q}}_P^N(\widetilde{\widetilde{\alpha}}^*) \geq \widetilde{\widetilde{q}}_R^N(\widetilde{\widetilde{\alpha}}^*)$, then total surplus maximizing level of transmission capacity would be $K = K(\widetilde{\widetilde{\alpha}}^*)$ when $C(K) = 0$; so, when $C(K) > 0$ the optimal transmission capacity will be less than $K(\overline{\alpha}^*)$ in this case.

5 Discussion of Results and Extensions

In this paper we analyzed the optimal regulatory policy in the context of a public/private mixed oligopolistic wholesale electricity market. The only regulatory tool at hand is the choice of the public firm's objective and there are no restrictions on the operation of the private firm. Our preliminary findings show that the optimal regulatory policy in both the two-node (no loop flow) network and the three-node network (with loop flows) will depend on whether the regulated firm or the private firm has marginal cost advantage in production, as well as on the capacity of the line linking the two sites.

Our results indicate that in many instances it will not be total surplus maximizing to instruct the regulated firm to maximize total surplus. However, except only one case, it is not optimal to instruct the regulated firm to maximize profits only.

First, we studied a two-node network in which the less efficient generator is located in the South where the demand is and the more efficient generator is located in the North (where there is no demand). When it is the private firm which is in the South, the optimal regulatory policy depends on the capacity of the link, K. If K is sufficiently high, then the public firm is instructed to maximize, not welfare or profits, but a strict convex combination of the two, given that shadow cost of public funds is low enough. When K is below a threshold level, then any objective function that gives a sufficiently high weight to welfare so as to cause the line to be congested, is part of the optimal regulatory policy, including pure total surplus maximization.

When it is the regulated firm which is in the South, provided that the line capacity K is sufficiently high, the optimal regulatory policy never involves pure total surplus maximization. The weight given to welfare in the objective function increases with the shadow cost of public funds and decreases with the efficiency gap between two firms. However, profits are always part of the objective function of the regulated firm and under some parameter values, the only part. The intuition behind this result is that since the public firm is not constrained by the transmission link, instructing it to give much emphasis to total social surplus results in overproduction. Due to its relative inefficiency and increasing marginal cost technology, it may produce where losses from cost inefficiencies outweigh consumer surplus gains. On the other hand, when transmission capacity is scarce and thus the line is congested in equilibrium, the public firm is instructed either to maximize total social surplus only, or a strict mix of profits and total social surplus, depending on the parameter values. However, in this case pure profit maximization is not part of the optimal regulatory policy.

Second, we looked at a three-node network where generators are located in different nodes and consumers are located on the remaining node. When the private firm is less efficient, provided that K is sufficently high, the optimal regulatory policy is to give most, and in some cases all, of the weight to total social surplus maximixation, and under no circumstance is pure profit maximization the optimal instruction. When the private firm is the more efficient and K is sufficiently high, the optimal regulatory policy is exactly the same as the optimal regulatory policy under a two-node network with an efficient private firm and K high enough.

An extension that we will consider is the assignment of property rights on the transmission network to the regulated or the private firm, or to an Independent System Operator, in a mixed electricity market. In this paper we assumed the transmission network to be a congestible free public good subject to only safe line flow limits. This extension will allow a more thorough study of the investment decisions relating to the choice of capacity K of the transmission line, which we treated only in one case in this study.

There are two other possible lines of research that will be pursued. First, other possible competition configurations in the electricity industry (e.g. Stackelberg competition where either the private or the regulated firm is the leader, Bertrand competition or competition on supply functions) can be considered. It is established in the literature that in standard industry structures without externalities, Stackelberg competition with the public firm as the leader Pareto dominates the Cournot competition. An immediate inquiry would be whether this result would stand in the electricity markets despite their unique characteristics, namely, transmission congestion and externalities stemming from loop flows. Second, informational asymmetries between the regulator and the regulated (public) firm should be considered in a more general analysis.

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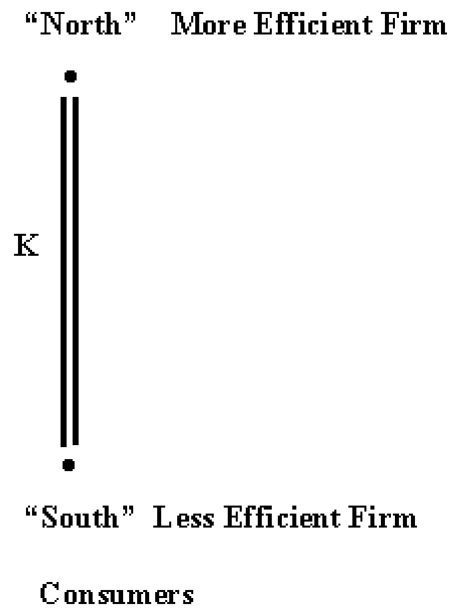

Figure 1: Two-Node (No Loop Flow) Network

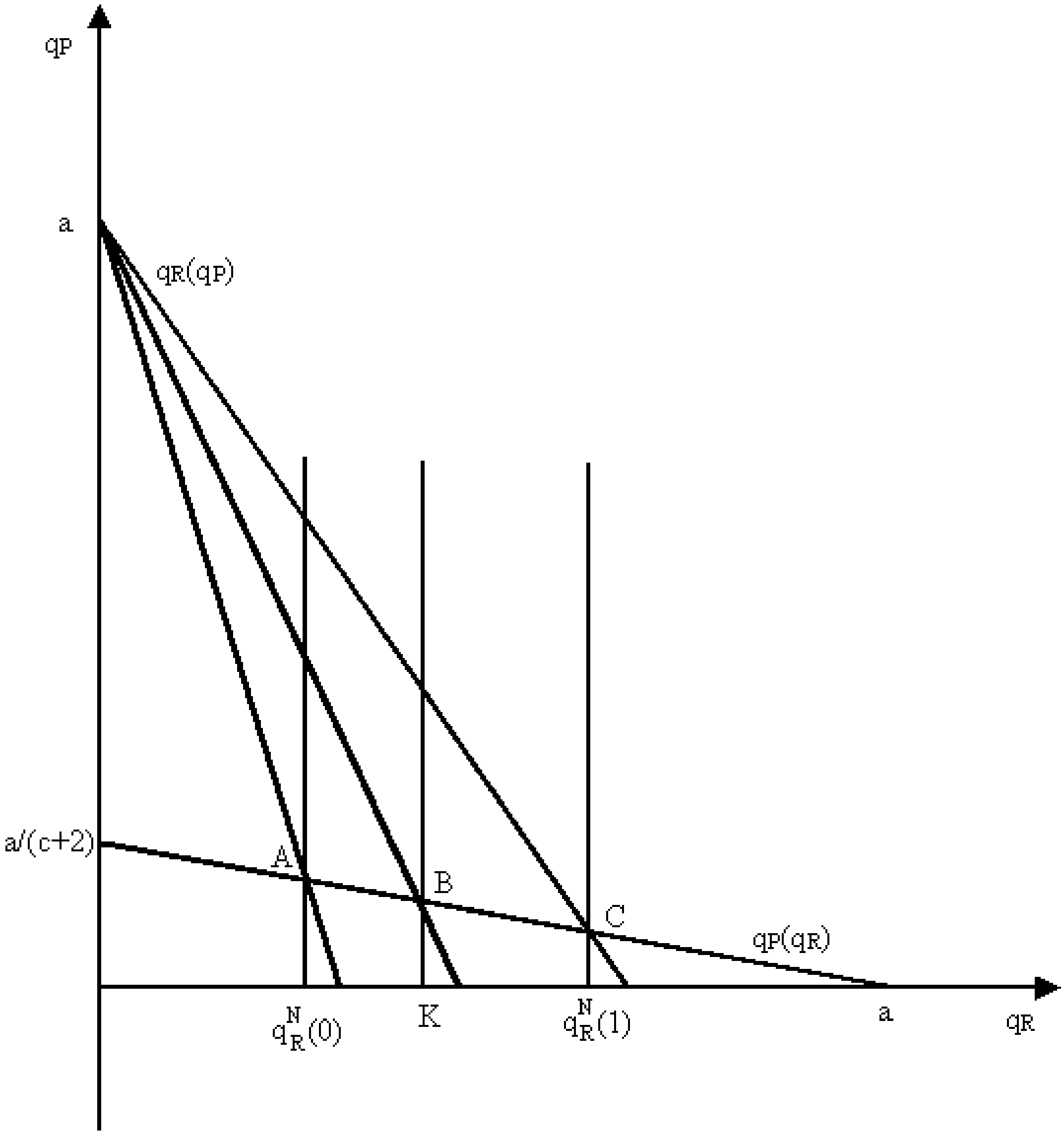

Figure 2: Equilibrium in Two-Node Network, $c_P = c$ and $c_R = 1$

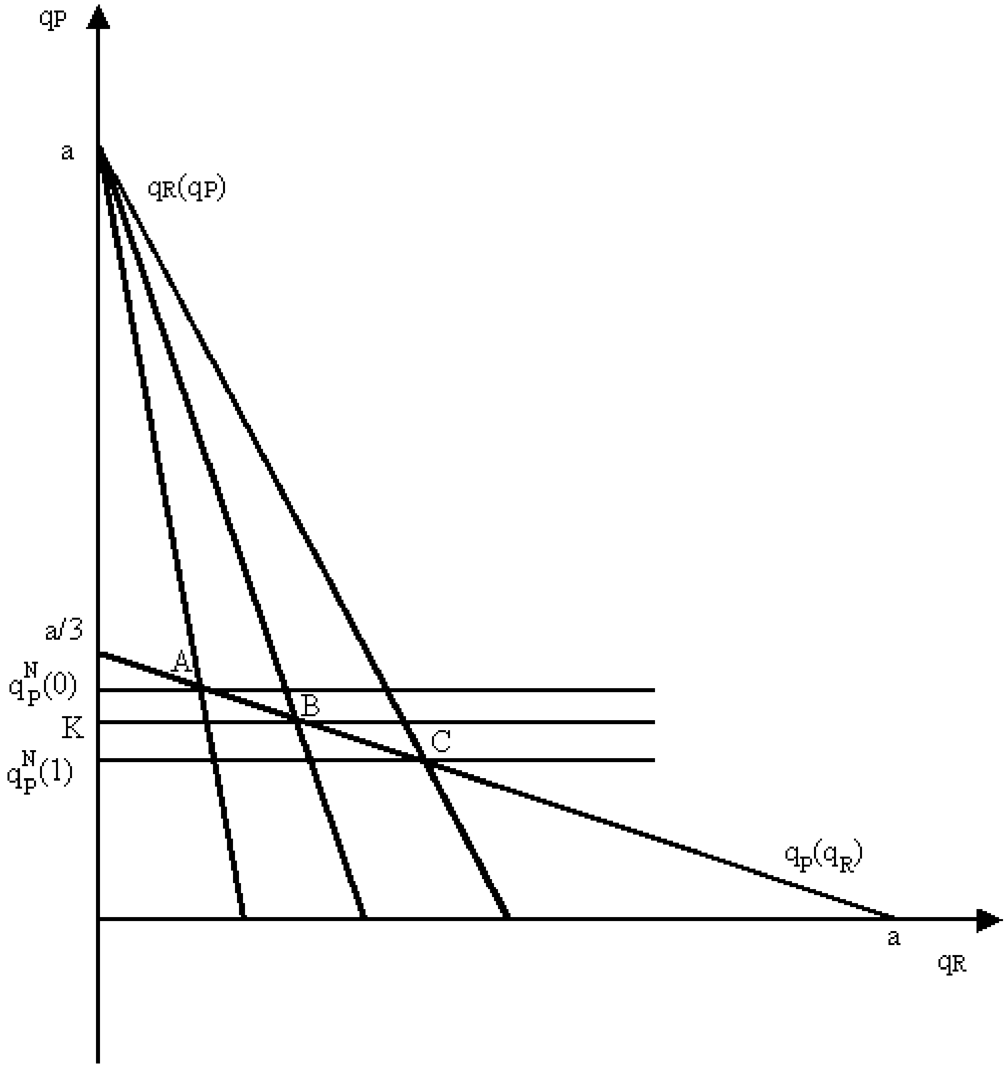

Figure 3: Two-Node Network, $c_P = 1$ and $c_R = c$

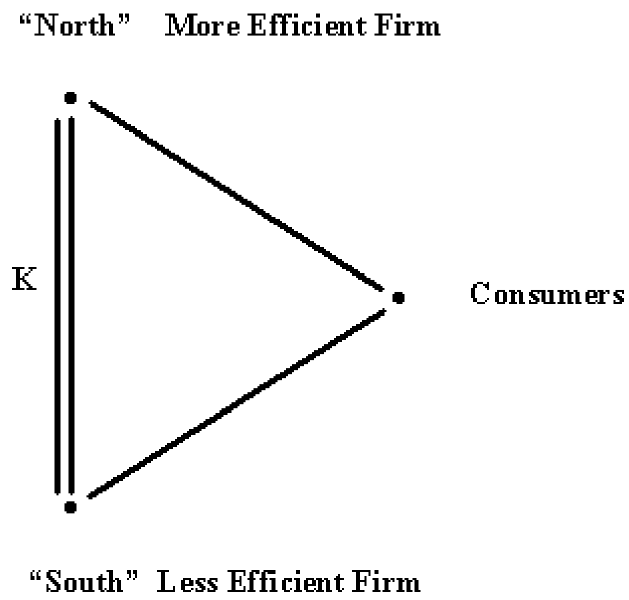

Figure 4: Three-Node (Loop Flow) Network

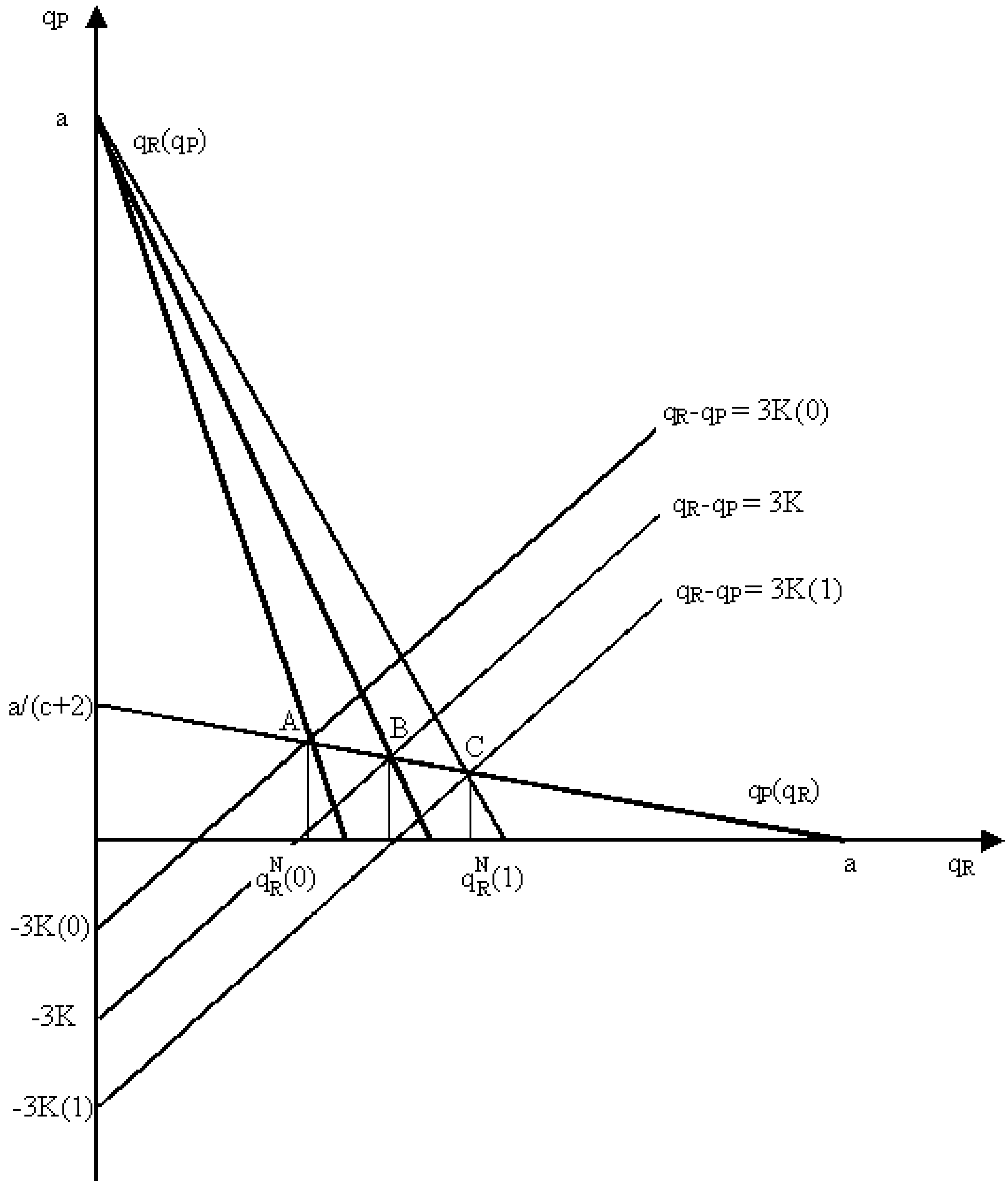

Figure 5: Three-Node Network, $c_P = c$ and $c_R = 1$

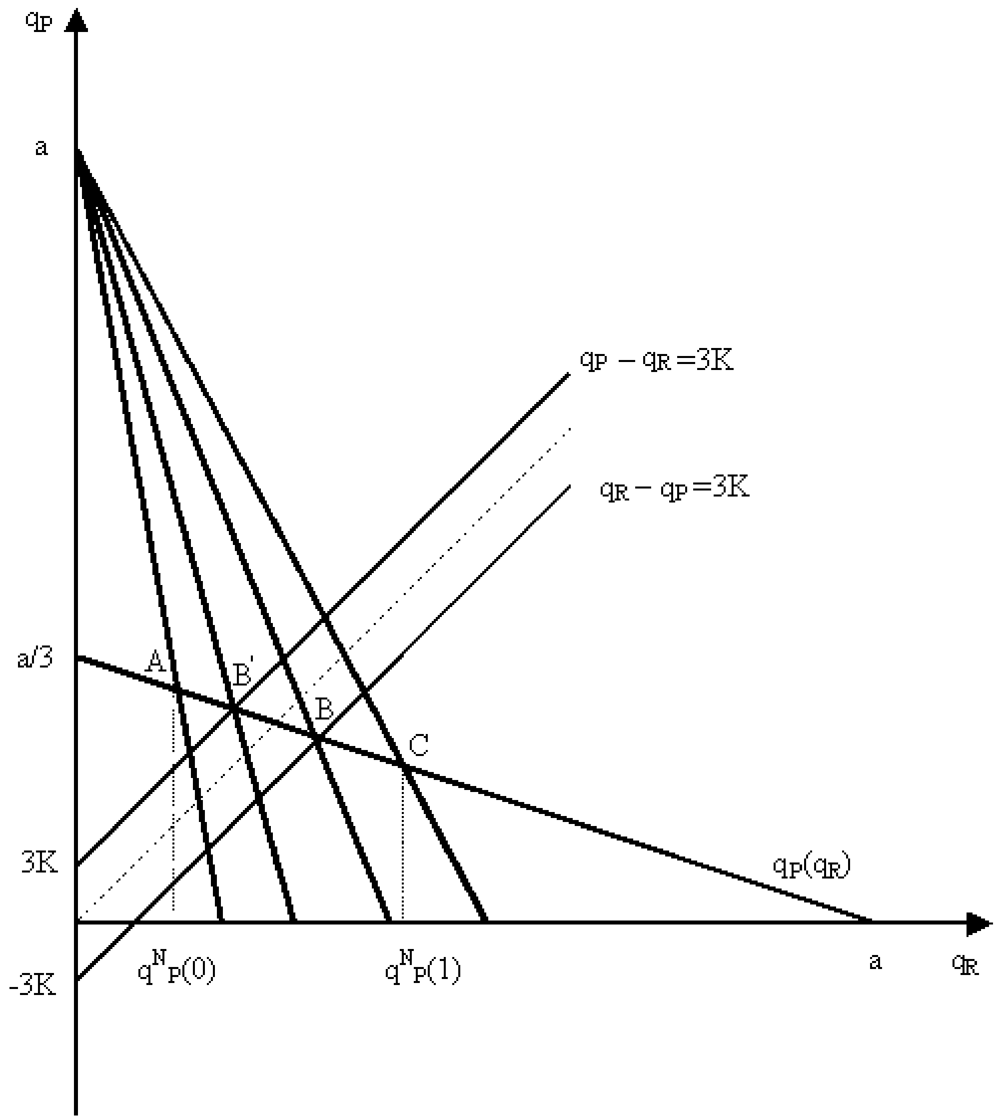

Figure 6: Three-Node Network $c_P = 1$ and $c_R = c$